May 28 Youtube: "Pascal's triangle for binomial expansion" by Khan Academy

MCR3U
Ms. Kueh

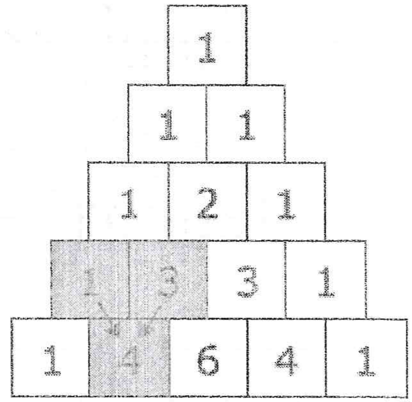
Pascal's Triangle

Unit Test on Friday, June 5

One of the most interesting Number Patterns is Pascal's Triangle (named after *Blaise Pascal*, a famous French Mathematician and Philosopher).

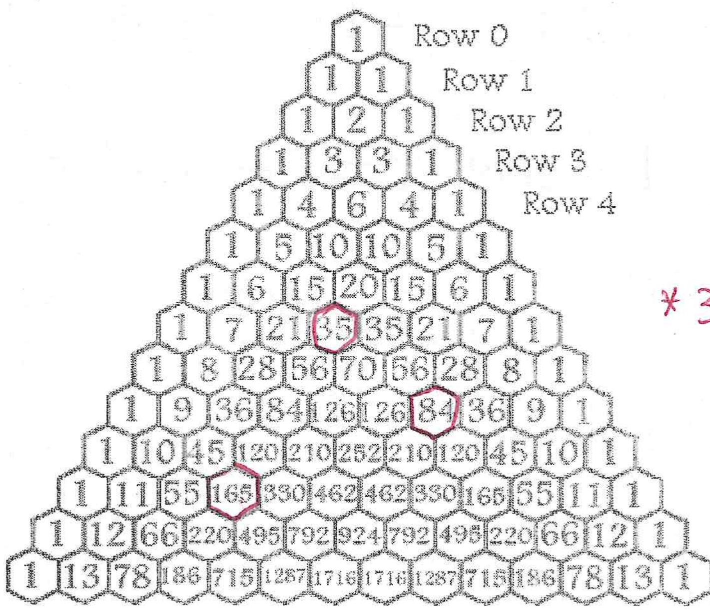
To build the triangle, start with "1" at the top, then continue placing numbers below it in a triangular pattern.

Each number is just the two numbers above it added together (except for the edges, which are all "1").



(Here I have highlighted that 1+3 = 4)

Identifying Terms by Position



*35 is $t_{7,3}$

*165 is $t_{11,3}$

Any term in Pascal's triangle can be identified by its position.

Eg. In row 3, the terms are 1, 3, 3, 1.

1 is $t_{3,0}$

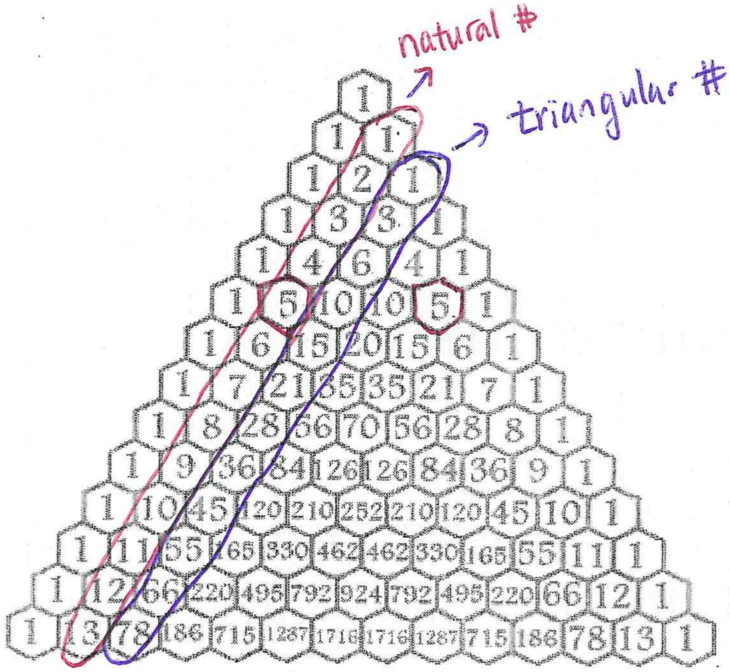
3 is $t_{3,1}$

3 is $t_{3,2}$

1 is $t_{3,3}$

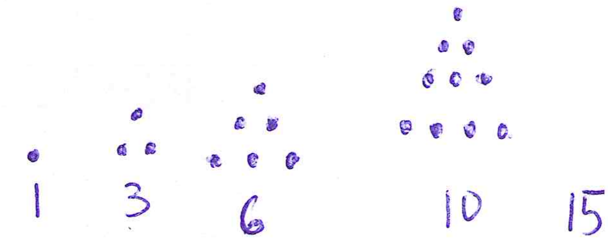
$t_{3,0}$ — 3: row # (y coordinate); 0: column # (x coordinate)

*84 is $t_{9,6}$



$t_{5,4} = {}_5C_4$

nCr



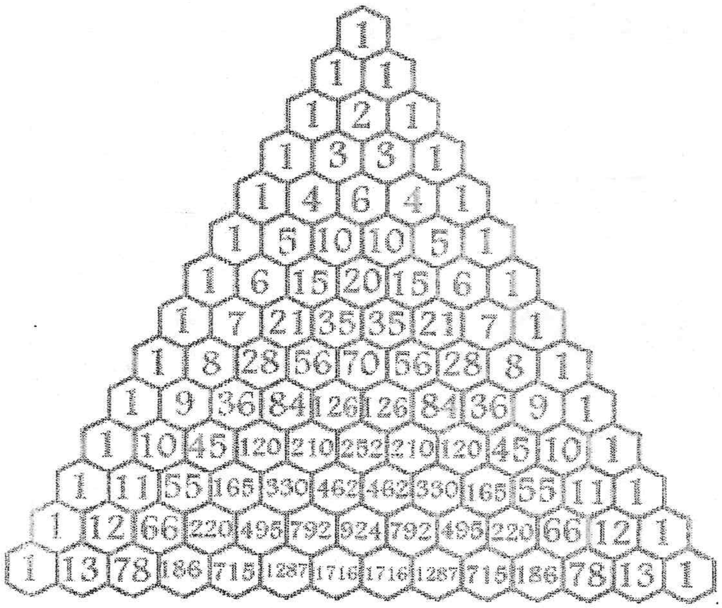
$t_{5,4} \rightarrow 5$

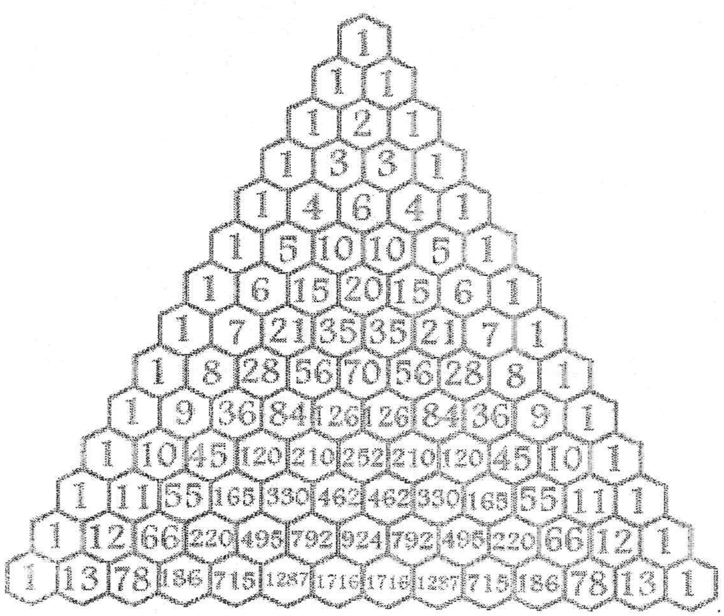
${}_5C_4 \rightarrow 5$

$t_{12,6} \rightarrow 924$

${}_{12}C_6 \rightarrow 924$

You will love this next year in MDM4U!





1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1
1 11 55 165 330 462 462 330 165 55 11 1
1 12 66 220 495 792 924 792 495 220 66 12 1
1 13 78 186 715 1287 1716 1716 1287 715 186 78 13 1

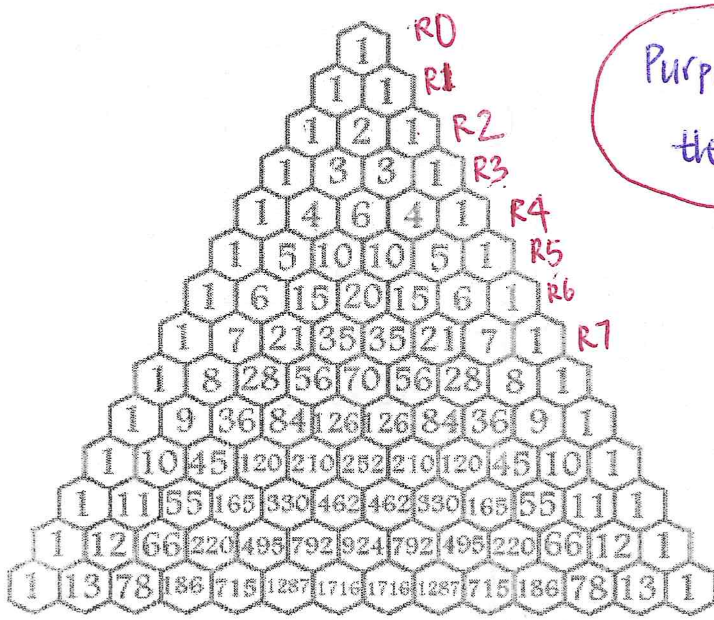
1 R $0 = 2^0$
1 1 R $1 = 2^1 = 2$
1 2 1 R $2 = 2^2 = 4$
1 3 3 1 $3 = 2^3 = 8$
1 4 6 4 1 $4 = 2^4 = 16$
1 5 10 10 5 1 5
1 6 15 20 15 6 1 6
1 7 21 35 35 21 7 1 7
1 8 28 56 70 56 28 8 1 8
1 9 36 84 126 126 84 36 9 1 9 → 2^9
1 10 45 120 210 252 210 120 45 10 1
1 11 55 165 330 462 462 330 165 55 11 1
1 12 66 220 495 792 924 792 495 220 66 12 1
1 13 78 186 715 1287 1716 1716 1287 715 186 78 13 1

<u>Row</u>

sum of each row $n = 2^n$

e.g) sum of all numbers of row 9 ⇒ $512 = 2^9$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1
1 11 55 165 330 462 462 330 165 55 11 1
1 12 66 220 495 792 924 792 495 220 66 12 1
1 13 78 186 715 1287 1716 1716 1287 715 186 78 13 1



Purpose: Pascal's triangle predicts the coefficients of expanded binomials.

Expanding Binomials:

$(x+1) =$ $x+1$

$(x+1)^2 =$ $(x+1)(x+1)$

$= x^2 + 2x + 1 \rightarrow$ (R2)

$(x+1)^3 =$ $(x+1)^2(x+1)$

$= (x^2+2x+1)(x+1)$

$= x^3 + 3x^2 + 3x + 1$ (R3)

1 3 3 1

$(x+1)^4 =$

(R4) 1 4 6 4 1

$= x^4 + 4x^3 1^1 + 6x^2 1^2 + 4x^1 1^3 + 1^4$

$= x^4 + 4x^3 + 6x^2 + 4x + 1$

$(a+b) =$ $a+b$

$(a+b)^2 =$ $a^2 + 2ab + b^2$ (R2)

1 2 1

$(a+b)^3 =$ $(a+b)^2 \cdot (a+b)$

$= (a^2+2ab+b^2)(a+b)$

$= a^3 + 2a^2b + ab^2 + a^2b + 2ab^2 + b^3$

$= a^3 + 3a^2b + 3ab^2 + b^3$

1 3 3 1 (R3)

$(a+b)^4 =$

(R4) 1 4 6 4 1

$a^4 + 4a^3b + 6a^2b^2 + 4a^1b^3 + b^4$

Using Pascal's Triangle, find

$(x+1)^7 =$ $x^7 + 7x^6 + 21x^5 + 35x^4 + 35x^3 + 21x^2 + 7x + 1$

1 7 21 35 35 21 7 1

Trickier:

$(x+2)^5 =$ $x^5 + (5x^4 \cdot 2) + (10x^3 \cdot 2^2) + (10x^2 \cdot 2^3) + (5x^1 \cdot 2^4) + 2^5$

1 5 10 10 5 1

$x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32$

Homework: pg. Pg. 377 C1, C3, C4, 1, 2, (3-8)ac, 10, 13, 16, 17

$(3x-2)^6 = (3x)^6 + 6(3x)^5 \cdot (-2)^1 + 15(3x)^4 \cdot (-2)^2 + 20(3x)^3 \cdot (-2)^3 + 15(3x)^2 \cdot (-2$

1 6 15 20 15 6 1

$6(3x)^1 \cdot (-2)^5 + 1 \cdot (-2)^6$

$= 3^6 \cdot x^6 + (-12 \cdot (3^5 \cdot x^5)) + 60(3^4 \cdot x^4) + -160(3^3 x^3)$

$+ 240(3^2 x^2) + (-192 \cdot 3x) + 64$

$= 729x^6 - 2916x^5 + 4860x^4 - 4320x^3 + 2160x^2$

$- 576x + 64$