May 29 MCR3U Park

* Unit 7 test will be Friday June 5

Arithmetic and Geometric Sequences

The following sequences are **Arithmetic Sequences:**	Find an explicit formula for t_n (or $f(n)$):
$t_n = a + (n-1)d$	
n= 1 2 3 4 5 1, 4, 7, 10, 13, ... +3 +3 +3 * Recursive Formula $t_n = t_{n-1} + 3$, $t_1 = 1$	$t_n = 1 + 3(n-1)$ $t_n = 1 + 3n - 3$ $t_n = 3n - 2$
n= 1 2 3 4 5 6 -5, -4, -3, -2, -1, 0, 1, ... +1 +1 +1 * Recursive Formula $t_n = t_{n-1} - 10$, $t_1 = 15$ 15, 5, −5, −15, −25, ... −10 −10 −10 −10	$t_n = -5 + 1(n-1)$ $t_n = -5 + n - 1$ $\therefore t_n = n - 6$ $t_n = 15 + [-10(n-1)]$ $\therefore t_n = -10n + 25$
The following sequences are Geometric Sequences:	Find an explicit formula t_n (or $f(n)$):
n 1 2 3 4 5 6 7 1, 2, 4, 8, 16, 32, 64, ... x2 x2 x2 x2 * Recursive Formula $t_n = t_{n-1} \times 2$, $t_1 = 1$	$t_n = a(r)^{n-1}$ $t_n = 1(2)^{n-1}$ $\therefore t_n = (2)^{n-1}$
* RF: $t_n = t_{n-1} \times 2$, $t_1 = 7$ 1 2 3 4 5 7, 14, 28, 56, 112, ... x2 x2 x2	$t_n = 7(2)^{n-1}$
* RF: $t_n = t_{n-1} \times 3$, $t_1 = -6$ 1 2 3 4 −6, −18, −54, −162, ... x3 x3	$t_n = -6(3)^{n-1}$

$8, -4, 2, -1, \frac{1}{2}, -\frac{1}{4}, \ldots$

$\times -\frac{1}{2}$ $\times -\frac{1}{2}$

$t_n = 8\left(-\frac{1}{2}\right)^{n-1}$

or

$t_n = \frac{8}{(-2)^{n-1}}$

$11, 22, 44, 88, \ldots$

$\times 2$ $\times 2$

$t_n = 11(2)^{n-1}$

Arithmetic Sequences

a) How can you tell when a sequence is Arithmetic?

def

is a sequence where the (difference) between consecutive terms is a constant.

add or minus

b) If a is the first term and d is common difference, what is the general formula t_n?

common difference ↓ difference between consecutive terms

$t_n = a + (n-1)d$ (Explicit Form)

a ↓ first term

d ↓ common difference

n ↓ term number (= input)

Geometric Sequences → formula: $t_n = a(r)^{n-1}$

r ↓ common ratio

a) How can you tell when a sequence is Geometric?

def

is a sequence in which the (ratio) of consecutive terms is a constant.

b) If a is the first term and r is common ratio, what is the general formula t_n?

common ratio ↓ ratio between consecutive terms

Example 1 Is the following sequence arithmetic or geometric?

-13, -19, -25, ...

-6 -6 → Arithmetic Sequence

AS → $t_n = a + (n-1)d$ (explicit formula)

a) Determine an explicit formula for the general term of the sequence.

$t_n = -13 - 6(n-1) = -6n - 7$

b) Determine a recursive formula for the general term of the sequence.

$t_n = t_{n-1} - 6$, $t_1 = -13$

c) What is the value of the 15th term of the sequence?

$t_{15} = -13 - 6(15-1) = -13 - (6 \cdot 14) = -13 - 84$

$= -97$

Example 2 Determine the 25th term of the sequence $-17, -10, -3, 4, \ldots$. $t_{25} = ?$

Arithmetic sequence: $t_n = a + d(n-1)$ (+7, +7)

$t_n = -17 + 7(n-1)$

$t_n = -17 + 7n - 7$

$\therefore t_n = 7n - 24$

$t_n = 7(25) - 24$ $\therefore t_{25} = 151$

$= 175 - 24$

$= 151$

Example 3 Find the number of terms in the sequence $3, 15, 27, \ldots, 495$. $n = ?$

(+12)

Arithmetic seq: $t_n = 3 + 12(n-1)$

$495 = 3 + 12n - 12$ $\therefore n = 42$

$495 + 9 = 12n$

$\frac{504}{12} = n$

Example 4 In an arithmetic sequence, $t_{12} = 52$ and $t_{22} = 102$, determine the first 3 terms.

Example 5 In a sequence, the common difference is 5, and $t_{12} = 62$. Is this sequence arithmetic or geometric? Find the first term of the sequence, and then write an explicit formula for the general term of the sequence.

Example 6 Is the following sequence arithmetic or geometric? Calculate the value of the 12^{th} term.

$$72, 48, 32, \ldots$$

Example 7 A sequence has a first term of 5, and a common ratio of -2.

a) Is this sequence arithmetic or geometric? Write the first 4 terms of the sequence.

b) If $t_n = 20\,480$, find n.

Example 8 Determine the number of terms in the sequence

$$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots, \frac{1}{1024}$$