

$$AS \rightarrow t_n = a + (n-1)d \quad (\text{explicit formula})$$

a) Determine an explicit formula for the general term of the sequence.

$$t_n = -13 - 6(n-1) = -6n - 7$$

b) Determine a recursive formula for the general term of the sequence.

$$t_n = t_{n-1} - 6, \quad t_1 = -13$$

c) What is the value of the 15<sup>th</sup> term of the sequence?

$$\begin{aligned} t_{15} &= -13 - 6(15-1) = -13 - (6 \cdot 14) = -13 - 84 \\ &= -97 \end{aligned}$$

**Example 2** Determine the 25<sup>th</sup> term of the sequence  $-17, -10, -3, 4, \dots$ .  $t_{25} = ?$

Arithmetic sequence:  $t_n = a + d(n-1)$   $+7$   $+7$

$$t_n = -17 + 7(n-1)$$

$$t_n = -17 + 7n - 7$$

$$\therefore t_n = 7n - 24$$

$$\begin{aligned} t_n &= 7(25) - 24 \quad \therefore t_{25} = 151 \\ &= 175 - 24 \\ &= 151 \end{aligned}$$

**Example 3** Find the number of terms in the sequence  $3, 15, 27, \dots, 495$ .  $n = ?$

Arithmetic Seq:  $t_n = 3 + 12(n-1)$   $+12$

$$495 = 3 + 12n - 12$$

$$\therefore n = 42$$

$$495 + 9 = 12n$$

$$\frac{504}{12} = n$$

June 1st

**Example 4** In an arithmetic sequence,  $t_{12} = 52$  and  $t_{22} = 102$ , determine the first

3

terms.

$$52 = a + (12-1)d \rightarrow 52 = a + 11d \quad \text{--- ①}$$

$$* \text{Sub } d=5 \rightarrow \text{①}$$

$$102 = a + (22-1)d \rightarrow 102 = a + 21d \quad \text{--- ②}$$

$$52 = a + 11(5)$$

$$52 = a + 55$$

$$-50 = -10d$$

$$52 - 55 = a \quad \therefore a = -3$$

$$5 = d$$

**Example 5** In a sequence, the common difference is 5, and  $t_{12} = 62$ . Is this sequence arithmetic or geometric? Find the first term of the sequence, and then write an explicit formula for the general term of the sequence.

$$t_n = -3 + (n-1)5$$

$\therefore$  First term is  $-3$ .

$\therefore$  3 terms are  $-3, 2, 7$ .

$d=5 \rightarrow$  Arithmetic sequence

\* QUIZ on WEDNESDAY! Test on Monday (June 8)

$$t_n = a + d(n-1)$$

$$62 = a + 5(12-1)$$

$$62 = a + 5(11)$$

$$62 = a + 55$$

$$62 - 55 = a \quad \therefore a = 7$$

Explicit  
∴ Formula is

$$t_n = 7 + 5(n-1)$$

$$t_n = 7 + 5n - 5$$

$$\therefore t_n = 5n + 2$$

∴ First term is 7.

**Example 6** Is the following sequence arithmetic or geometric? Calculate the value of the 12<sup>th</sup> term.

$$* \frac{2}{3} = \frac{t_{n+1}}{t_n} = \frac{48}{72} \xrightarrow{\div 24} \frac{2}{3}$$

$$72, 48, 32, \dots$$

$\xrightarrow{-24} \xrightarrow{-16}$  → There is no common difference  
↳ not arithmetic sequence

$$* \frac{2}{3} \times \frac{2}{3} \rightarrow r = \frac{2}{3} \rightarrow \text{Geometric sequence}$$

$$* \text{GS formula} \Rightarrow t_n = a(r)^{n-1}$$

$$t_n = 72 \left(\frac{2}{3}\right)^{n-1}$$

$$t_{12} = 72 \left(\frac{2}{3}\right)^{12-1}$$

$$\rightarrow t_{12} = 72 \left(\frac{2}{3}\right)^{11}$$

$$\therefore t_{12} = 0.83$$

**Example 7** A sequence has a first term of 5 and a common ratio of -2 →  $r = -2 \rightarrow \text{GS}$

a) Is this sequence arithmetic or geometric? Write the first 4 terms of the sequence.

Formula is:

$$\therefore t_n = 5(-2)^{n-1}$$

$$\begin{matrix} \times -2 & \times -2 & \times -2 \\ \downarrow & \downarrow & \downarrow \end{matrix}$$

∴ First 4 terms are: 5, -10, 20, -40

b) If  $t_n = 20480$ , find  $n$ .

$$t_n = 5(-2)^{n-1}$$

$$\frac{20480}{5} = \frac{5(-2)^{n-1}}{5}$$

$$4096 = (-2)^{n-1}$$

$$4096 = (-2)^{12}$$

$$\rightarrow n-1 = 12$$

$$\therefore n = 13$$

**Example 8** Determine the number of terms in the sequence

$$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots, \frac{1}{1024} \rightarrow n = ?$$

$$\frac{1}{4} \div \frac{1}{2} = \frac{1}{4} \times 2 = \frac{1}{2}$$

$$\times \frac{1}{2} \quad \times \frac{1}{2}$$

$$\rightarrow r = \frac{1}{2} \rightarrow t_n = \frac{1}{2} \left(\frac{1}{2}\right)^{n-1}$$

(G Sequence)

$$2 \times \frac{1}{1024} = \left(\frac{1}{2}\right) \cdot \left(\frac{1}{2}\right)^{n-1} \times 2$$

$$\frac{2}{1024} = \left(\frac{1}{2}\right)^{n-1}$$

$$0.001953 = (0.5)^9$$

$$n-1 = 9 \rightarrow n = 10$$

→ Using calculator  
Trial and error

∴  $\frac{1}{1024}$  is 10th term.

**Example 9** In a geometric sequence  $t_5 = 3$  and  $t_{14} = 1536$ . Determine the general formula for the sequence. What is the value of  $t_9$ ?

$$t_n = a(r)^{n-1}$$

$$3 = a(r)^{5-1} \rightarrow 3 = a(r)^4 \rightarrow \frac{3}{r^4} = a \quad \text{--- (1)}$$

$$1536 = a(r)^{14-1} \rightarrow 1536 = a(r)^{13} \quad \text{--- (2)}$$

Sub eq (1) into (2)

$$\frac{1}{3} \times 1536 = \frac{3}{r^4} (r)^{13} \times \frac{1}{3}$$

$$512 = \frac{r^{13}}{r^4}$$

$$512 = r^{13-4}$$

$$512 = r^9$$

$$\sqrt[9]{512} = r$$

$$\therefore r = 2$$

Sub  $r=2$  into (1)

$$\therefore t_n = \frac{3}{16} (2)^{n-1}$$

$$\frac{3}{2^4} = a \quad \therefore a = \frac{3}{16}$$

$$t_9 = \frac{3}{16} (2)^{9-1} \rightarrow \therefore t_9 = 48$$

**Example 10** After being dropped from a height of  $1\text{ m}$ , a ball bounces off each time to 75% of its previous height. What maximum height will the ball reach after its 8<sup>th</sup> bounce?

**Homework:** Pg. 385 C1, C2, #1abe, 2ade, 3adfh, 6 7abc, 8, 9, 18, 19, 22  
Pg. 392 (1-5)ad, 6ab, 8, 9, 11, 16, 17, 20

## Arithmetic Series

**Warm up** If  $(x + 1)$ ,  $(-2x - 4)$ , and  $(x + 15)$  are three consecutive terms of an arithmetic sequence, determine the three terms.

An arithmetic series is the sum of the terms of an arithmetic sequence.

Eg. Sequence  $1, 3, 5, 7, \dots$

Series

**Developing the formula for the sum**

Find the sum of the first  $100$  natural numbers.

