$$2 \times \frac{1}{1024} = \left(\frac{1}{2}\right) \cdot \left(\frac{1}{2}\right)^{n-1} \times 2$$

$$\frac{2}{1024} = \left(\frac{1}{2}\right)^{n-1}$$

$$0.001953 = (0.5)^{9}$$ → using calculator
[trial and error

$\therefore \frac{1}{1024}$ is 10th term.

$$n-1 = 9 \rightarrow n = 10$$

Example 9 In a geometric sequence $t_5 = 3$ and $t_{14} = 1536$. Determine the general formula for the sequence. What is the value of t_9?

$$t_n = a(r)^{n-1}$$

$$3 = a(r)^{5-1} \rightarrow 3 = a(r)^4 \longrightarrow \frac{3}{r^4} = a \text{ — } ①$$

$$1536 = a(r)^{14-1} \rightarrow 1536 = a(r)^{13} \text{ — } ②$$

sub eq ① into ②

$$\frac{1}{3} \times 1536 = \frac{3}{r^4}(r)^{13} \times \frac{1}{3}$$

$$512 = \frac{r^{13}}{r^4}$$

$$512 = r^{13-4}$$

$$512 = r^9$$

$$\sqrt[9]{512} = r$$

$\therefore r = 2$ $\quad \therefore t_n = \frac{3}{16}(2)^{n-1}$

Sub $r=2$ into ① $\quad \frac{3}{2^4} = a \quad \therefore a = \frac{3}{16}$

$$t_9 = \frac{3}{16}(2)^{9-1} \rightarrow \therefore t_9 = 48$$

June 2

Example 10 After being dropped from a height of $1\ m$, a ball bounces off each time to 75% of its previous height. What maximum height will the ball reach "after its 8th bounce?" → $n=8$ → $t_8 = ?$

$a = 1m \xrightarrow{\times 0.75} 0.75m \xrightarrow{\times 0.75} 0.5625$ → GS → $t_n = a(r)^{n-1}$ → $t_n = 1(0.75)^{n-1}$

common ratio

→ when $n=8$ → $t_n = 1(0.75)^{8-1}$

$$t_n = 0.75^7 = 0.1335m$$

$\therefore$ The ball will reach 0.1335m after 8th bounce.

Homework: Pg. 385 C1, C2, #1abe, 2ade, 3adfh, 6 7abc, 8, 9, 18, 19, 22
Pg. 392 (1-5)ad, 6ab, 8, 9, 11, 16, 17, 20

* Quiz : Wed, June 3   * Test : Monday, June 8.

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Arithmetic Series

Youtube : "Finding the sum of a finite Arithmetic Series" by patrick JMT

Warm up If $(x+1)$, $(-2x-4)$, and $(x+15)$ are three consecutive terms of an arithmetic sequence, determine the three terms.

↳ common difference "d" should be found.

$x+1, \; -2x-4, \; x+15$

$d_1 = d_2$

$d_1 = -2x-4-(x+1)$
$d_1 = -2x-x-4-1$
$d_1 = -3x-5$

$d_2 = x+15-(-2x-$
$d_2 = x+2x+15+$
$d_2 = 3x+19$

$-3x-5 = 3x+19$
$-6x = 24$
$\therefore x = -4$

$\therefore$ 3 terms are $-3, 4, 11$

$= -4+1$

An arithmetic series is the sum of the terms of an arithmetic sequence.

Eg. Sequence $1, 3, 5, 7, \ldots$

Series : $1+3+5+7$

Developing the formula for the sum

sum

Find the sum of the first 100 natural numbers.

$$S_n = \frac{n}{2}(a+t_n) = \frac{100}{2}(1+100) = 50(101) = 5050$$

$$\therefore S_{100} = 5050$$

$$S_n = \frac{n}{2}[2a+(n-1)d] = \frac{100}{2}[2\cdot 1 + (100-1)1]$$

$$= 50[2+99] = 50(101) = 5050$$

* Arithmetic Series

* You are given the first term, the last term and the number of terms of an arithmetic series

t_n n

sum common difference

$$\boxed{S_n = \frac{n}{2}(a+t_n) \quad \text{or} \quad S_n = \frac{n}{2}[2a+(n-1)d]}$$

When you are given common difference, first term and the number of terms

Example 1

Find the sum of the first 10 terms of the series $-1 + 1 + 3 + 5 + \cdots$

$= n$

$+2 \quad +2 \quad +2$

$$S_{10} = \frac{n}{2}(2a + d(n-1))$$

$$S_{10} = \frac{10}{2}(2(-1) + 2(10-1))$$

$$= 5(-2 + 2(9)) = 5(16) = 80$$

$\therefore S_{10} = 80$

Example 2

Find the sum of the arithmetic series $-8 + (-5) + (-2) + \ldots + 139$.

$+3 \quad +3 = d \qquad n = ?$

In order to find n, we will use arithmetic sequence formula:

$$t_n = a + d(n-1)$$

$$139 = -8 + 3(n-1)$$

$$139 = -8 + 3n - 3$$

$$139 + 11 = 3n$$

$$150 = 3n$$

$$n = \frac{150}{3} = 50$$

$$S_{50} = \frac{50}{2}[2 \cdot (-8) + (50-1) \cdot 3]$$

$$= 25[-16 + 147]$$

$$= 25[131]$$

$$= 3275$$

$\therefore S_{50} = 3275$

Example 3

Find the sum of the first 25 terms of an arithmetic series if $t_{12} = -26$ and $t_{22} = -46$. $\rightarrow n = 25, \; a = ?, \; d = ?, \; t_n = ?$

In order to find d, let's use AS formula:

$$t_n = a + d(n-1)$$

$$-26 = a + d(12-1) \quad \text{by using } t_{12} = -26$$

$$-26 = a + 11d$$

$$-26 - 11d = a \quad \text{— ①}$$

By using $t_{22} = -46$

$$-46 = a + d(22-1)$$

$$-46 = a + 21d$$

$$-46 - 21d = a \quad \text{— ②}$$

Let's set ① = ②

$$-26 - 11d = -46 - 21d$$

$$-11d + 21d = -46 + 26$$

$$\frac{10d}{10} = \frac{-20}{10}$$

$$d = -2 \rightarrow \text{sub into ①}$$

$$a = -26 - 11(-2) = -4$$

$$S_{25} = \frac{25}{2}[2(-4) + (25-1) \cdot (-2)]$$

$$= \frac{25}{2}[-8 + (-48)]$$

$$= \frac{-1400}{2} = -700$$

$\therefore S_{25} = -700$

Homework: pg. 399 #C2, 1EOO, 2aef, 3ac, 4ac, 5d, 6, 7, 8ac, 9, 11, 17

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Geometric Series

Andrew's neighbours are planning on taking a vacation for two weeks. They have asked him to look after their cat and to water their plants. The neighbours have offered to pay him \$5 per day or \$0.01 the first day, \$0.02 the second day, \$0.04 the third day, \$0.08 the fourth day, etc.

1. Which method of payment should Andrew choose?

2. Andrew notices that the amounts he will earn each day if he chooses the second plan are the terms of the geometric sequence
   0.01 , $0.01(2)^1$, $0.01(2)^2$, $0.01(2)^3$,..., $0.01(2)^{13}$
   Calculate the amount that he will have at the end of two weeks.

$$S_{14} = 0.01 + 0.01(2) + 0.01(2)^2 + 0.01(2)^3 + \cdots + 0.01(2)^{13}$$

Generalize this method to find a formula for the geometric series
$S_n = a + ar + ar^2 + \cdots + ar^{n-1}$