MCR3U
Ms. Kueh

Geometric Series

* Test: Monday, June 8

Andrew's neighbours are planning on taking a vacation for two weeks. They have asked him to look after their cat and to water their plants. The neighbours have offered to pay him $5 per day or $0.01 the first day, $0.02 the second day, $0.04 the third day, $0.08 the fourth day, etc.

1. Which method of payment should Andrew choose?

$\frac{\$5}{\text{day}} \times 14 \text{ days} = 70\,\$$ or

2. Andrew notices that the amounts he will earn each day if he chooses the second plan are the terms of the geometric sequence
   $0.01, 0.01(2)^1, 0.01(2)^2, 0.01(2)^3, \ldots, 0.01(2)^{13}$
   Calculate the amount that he will have at the end of two weeks.

$81.92

$$S_{14} = 0.01 + 0.01(2) + 0.01(2)^2 + 0.01(2)^3 + \cdots + 0.01(2)^{13}$$

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$$S_n = \frac{a(r^n - 1)}{r-1}$$

$$S_{14} = \frac{0.01(2^{14} - 1)}{2-1} = \frac{163.83}{1} = 163.83\,\$$$

Generalize this method to find a formula for the geometric series
$S_n = a + ar + ar^2 + \cdots + ar^{n-1}$

Geometric series is sum of the terms in a geometric sequence.

e.g) $-3 + 6 - 12 + 24 \ldots\ldots$

Formula: $S_n = \frac{a(r^n - 1)}{r-1}$, $r \neq 1$

a = first term r = common ratio

Example 1 Find the sum of the first ten numbers in the series. $S_{10} = ?$

a) $2 + 6 + 18 + \cdots$

$\times 3 \quad \times 3$ so $r = 3$

$$S_{10} = \frac{2(3^{10} - 1)}{3 - 1} = 59048$$

b) $-2 + 4 - 8 + \cdots$

$\times -2 \quad \times -2$ $r = -2$

$$S_{10} = \frac{-2((-2)^{10} - 1)}{-2 - 1}$$

$$= \frac{-2046}{-3} = 682$$

c) $1 + \frac{1}{3} + \frac{1}{9} + \cdots$

$\times \frac{1}{3} \quad \times \frac{1}{3}$ so $r = \frac{1}{3}$

$$S_{10} = \frac{1\left(\left(\frac{1}{3}\right)^{10} - 1\right)}{\left(\frac{1}{3} - 1\right)} = \frac{\left(\frac{1}{59049} - 1\right)}{\frac{(1 - 3)}{3}} = \frac{\frac{1 - 59049}{59049}}{\frac{-2}{3}} = \frac{\frac{-5904}{5904}}{\frac{-2}{3}}$$

$$\frac{-59048}{59049} \times \frac{3}{-2} = \frac{-177144}{-118098} = 1.5$$

Example 2 Find the sum of the series $\frac{1}{25} + \frac{1}{5} + 1 + \cdots + 3125 \rightarrow n = ?$

$\times 5 \quad \times 5$ so $r = 5$

In order to find n, we use G sequence formula: $t_n = a(r)^{n-1}$

$25 \times 3125 = \frac{1}{25}(5)^{n-1} \times 25$

$78125 = 5^{n-1}$

$78125 = 5^7 \rightarrow$ Trial and error by using calculator

$5^7 = 5^{n-1}$

$7 = n - 1$

$\therefore n = 8$

G series formula

$$S_8 = \frac{\frac{1}{25}(5^8 - 1)}{5 - 1}$$

$$= \frac{\frac{1}{25} \times 390624}{4}$$

$$= 3906.24$$

Homework: pg. 407 #C1, 1, 2ade, 3acf, 4ad, 5b, 6b, 8