MCR3U   **Applications of Sequences and Series**
Ms. Kueh

Determine the type of problem, then solve!

1. A tennis tournament has 128 entrants. A player is dropped from the tournament after losing one match. Winning players go on to another match. What is the total number of matches that will be played in this tournament?

final $n = ?$

final $t_n = 1$

n 1   2   3   4

$128, 64, 32, 16, \ldots$

$\times \frac{1}{2}$ $\times \frac{1}{2}$ → so $r = \frac{1}{2}$

Geometric sequence → $t_n = a(r)^{n-1}$

$t_n = 128\left(\frac{1}{2}\right)^{n-1}$

$1 = 128\left(\frac{1}{2}\right)^{n-1}$

$\dfrac{1}{128} = \dfrac{128\left(\frac{1}{2}\right)^{n-1}}{128}$

$\dfrac{1}{128} = \left(\frac{1}{2}\right)^{n-1}$

$\left(\frac{1}{2}\right)^{7} = \left(\frac{1}{2}\right)^{n-1}$

$7 = n-1$

$8 = n$

∴ Total number of matches will be 8 matches.

2. You see a great video on YouTube at $1\ pm$. You are the first person to see this video. It takes you 1 hour to show it to 10 of your friends. Each of your friends shows the video to 10 of their friends in the next hour. After another hour, each of those people shares it with another 10 people. The sharing of the video continues the same way. If a video is considered to be viral when it has 10 million views within an hour, how many hours will it take for the video to go viral?

✦↗
will not be
on the test

$S_n = 10,000,000$ $n = ?$

n 1   2   3   4

$1 + 10 + 100 + 1000 \ldots$

$\times 10$ $\times 10$ So $r = 10$

Geometric series

$S_n = \dfrac{a(r^n - 1)}{r-1}$

$S_n = \dfrac{10(10^n - 1)}{10-1} + 1$

Remember $r \neq 1$, so

We have to use "10" as the first term!

$10,000,000 = \dfrac{10(10^n - 1)}{9} + 1$

$\dfrac{9}{10} \times 9,999,999 = \dfrac{10(10^n-1)}{9} \times \dfrac{9}{10}$

$8999999.1 = 10^n - 1$

$9,000,000.1 = 10^n$

$8912509 = 10^{6.95}$

It takes 6.95 hours

3. A student is offered a job with a math teacher that will last 20 h. The first option pays $4.75 for the first hour, $5 for the second hour, $5.25 for the next hour, and so on.

The second option pays $\frac{\$7}{h}$ for all hours worked.

   a. Which option pays more? How much more?

Option B : $\frac{\$7}{hour} \times 20$ hours

Option B $= \$140$

$$\begin{array}{ccc} 1 & 2 & 3 \\ \$4.75 + & \$5 + & \$5.25 \end{array}$$

$+0.25 \quad +0.25 = d \rightarrow$ Arith series

$S_n = 10(9.5 + 4.75)$

$S_n = \dfrac{n}{2}(a + t_n)$

$S_n = 142.50$

$S_n = \dfrac{20}{2}(4.75 + t_n) \rightarrow$ can't do it!
because there are two unknown variables.

∴ The first option pays $2.50 more than second option.

$S_n = \dfrac{20}{2}(2 \cdot 4.75 + 0.25(20-1))$

$S_n = \dfrac{n}{2}(2a + (n-1)d)$

   b. In the first option, how much is the student paid in the 10th hour?

We should use arithematic sequence formula

$t_n = a + (n-1)d$

$t_{10} = 4.75 + (10-1)\cdot 0.25$

$t_{10} = 4.75 + 2.25$

$t_{10} = 7\$$

∴ The student is paid $7 at 10th hour.

Identify the type of question:

Arithmetic Sequence - $A\ t_n$

Arithmetic Series - $A\ S_n$

Geometric Sequence - $G\ t_n$

Geometric Series - $G\ S_n$

1. In a pyramid of dominoes, the top domino falls and hits the two dominoes in the second row. Those dominoes then topple, and each hit two other dominoes that are in the next row. (Ryan Henderson)

    a. If this pattern continues, how many dominoes in total have toppled by the 12th row?
    b. How many dominoes are toppled in the 12th row?

2. Jack brought 243 candies to school. Since he didn't really like them, he decided to give one-third of his candies to a friend he encounters. If he encountered 5 friends, how many candies would he have left? (Jack Kang)

3. The raptors are giving away free basketball tickets for the next big game against the Celtics. The general manager tells his son about this deal before he goes to school. The son then tells 6 of his friends. Each one of them tells 6 of their friends. This trend continues... By the end of the 6th round of people told; How many people at the son's school know about the free tickets? (Dad doesn't count as a round) (Graydon Sproule)

4. Ranvier is 6 years old. His parents decide to give him a $1.50 weekly allowance until he is 7 years old. They will increase his allowance by $0.75 cents each year. When he is 19 years old, how much will he get for his weekly allowance?

5. The modern summer Olympic Games were first held in Athens, Greece in 1896. The games were to be held every four years. What year was the 12th modern Olympic Game?

6. A virus that was found reproduces asexually by splitting itself in half. So it started off with one, then 2, then 4. How many of this species will there be on the 100th level or cycle? (Amina Benmassoud)

7. A disease is spreading through the city. The first person to receive the disease sneezes and passes it on to three other people, and each of them sneezes and passes it on to three more people. (Mariam Aly)

    a. How many people in total will have the disease by the 7th round of sneezing?
    b. How many people will get the disease in the 10th round of sneezing?

8. On January 1st, Zach opens a bank account, with an initial deposit of $100. On the 1st of each month, (January excluded), his grandparents will match whatever is in the account.

How much money will be in the account on Dec 31. of the same year? (Assuming Zach makes no more deposits) (Zach Loeb-Houston)

9. Avery is having a birthday party. Suppose that in the first round of invitations, Avery texts 8 people. Each invited person then texts 5 people and each of those people texts 5 people, and so on. What is the total number of people invited by the 8th round of texts? (Henry Choi)

10. A concert hall has 50 rows of seats. Each row has 4 more seats than the previous row. If the first row has 16 seats.

    a. How many seats are in the 30th row?
    b. How many seats are there in total?

11. Michelle is a marine biologist. She accepted a job that pays $46 850 in the first year and $56 650 in the eighth year.
    a. If her salary increases by the same amount each year, what will her salary be in the 5th year?
    b. What is the total amount that Michelle will earn after 10 years?

12. A chain email is sent to 100 people & all those people forward it to 4 other people & every person who receives the email forwards it to 4 more people. (Jacob Jablonski)

    a. after 10 rounds of emails, how many people have received the email?
    b. How many people received the email in the 10th round of emails?

Answers:

1. a) 4095   b) 2048
2.   32 candies
3.   9 331
4. $11.25
5.   1940
6.   6.338253001 x 10^{29}
7. a) 1 093   b) 19 683
8. $204 800
9.   781 248 people
10. a) 132   b) 5 700
11. a) $52, 450   b) $531 500
12. a) 34 952 500   b) 26 214 400

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1.
n	1	2	3	4
Domino	1	2	4	8

$\times 2 \quad \times 2 \quad \times 2 \quad$ so $r = 2$

b) $t_{12} = a(r)^{n-1}$

$= 1(2)^{12-1}$

$= 2^{11}$

$= 2048$

a) $S_{12} = ? \rightarrow$ G series formula

b) $t_{12} = ? \rightarrow$ G sequence formula

a) $S_{12} = \dfrac{a(r^n - 1)}{r-1}$

$= \dfrac{1(2^{12} - 1)}{2-1}$

$= \dfrac{4095}{1} = 4095$

2.
Friends		1	2	3	4	⑤ friends
n	1	2	3	4	5	6.
candies	243	162	108			$t_6 = ?$

$\times \frac{2}{3} \quad \times \frac{2}{3} \rightarrow r = \frac{2}{3}$

$t_6 = a(r)^{n-1}$ (G sequence)

$t_6 = 243\left(\frac{2}{3}\right)^{6-1}$

$t_6 = 243\left(\frac{2}{3}\right)^5$

$t_6 = 243 \cdot \dfrac{32}{243}$

$t_6 = 32$

∴ After encountering 5 friends, he would have 32 candies.

#3
Round	1	2	3
N	1	2	3

$1 + 6 + 36 +$

$\times 6 \quad \times 6 \quad \times 6 = r$

N = Round #, so $S_6 = ?$

G series formula

$S_6 = \dfrac{1(6^6 - 1)}{6-1} = \dfrac{46655}{5}$

$S_6 = 9331$

#4.

Age	6	7	8
N	1	2	3

$1.50 $2.25 $3.00

+ 0.75   + 0.75 = d

arithematic sequence

$t_n = a + (n-1)d$

$t_{14} = 1.50 + (14-1)0.75$

$t_{14} = 1.50 + 9.75$

$t_{14} = \$11.25$

* 19 years old $\Rightarrow N = ?$

$N = 19 - 5 = 14$

$t_{14} = ?$

∴ when he is 19 years old,

he will receive $11.25