

Simple and Compound Interest

GIC

\*D2L Quiz should be done by Sunday! (2.5 hour)

Guaranteed Investment Certificate is a form of investment that guarantees a certain rate of return. The minimum investment amount is usually \$500.00. Maturity periods may range from 30 days to 10 years. If you withdraw money before the term is over, you are not paid any interest and may even be required to pay a fee. GICs can be calculated with simple or compound interest. *(You can buy GIC at banks such as RBC, BMO and credit union)*

**Simple Interest**

Aayush deposits \$1000 into a GIC that earns 3% simple interest annually. Each year, the interest is paid out to him in cash.

Principal

– the original amount of a debt or investment on which interest is calculated.

a) How much is he paid each year?

$$\$1000 (0.03) = \$30 \text{ each year}$$

b) Determine the total amount of money he has after 1, 2, 3, and 4 years and calculate first differences.

Time (years)	Amount (\$)	First Differences
0	1000	
1	1030	30
2	1060	30
3	1090	30
4	1120	30

$$\text{Interest amount} = \text{Principal} \times \text{interest percentage}$$

c) Is this an arithmetic/geometric sequence/series?

arithmetic sequence

**Example 1** Jessica deposits \$500 into a guaranteed investment certificate (GIC) that earns simple interest every year. What annual rate of interest must be earned so that the investment doubles in 8 years?

$$\begin{array}{l}
 \$500, 0, 0 \dots \$1000 \\
 n \quad 1, 2, 3 \dots 8 \\
 \hline
 \frac{71.43}{500} \times 100 = 14.28\% \\
 \therefore \text{Interest rate is } 14.28\%
 \end{array}$$

$$\begin{aligned}
 t_n &= a + d(n-1) \\
 1000 &= 500 + d(8-1) \\
 500 &= 7d \\
 500 \div 7 &= d \quad \therefore d = \$71.43
 \end{aligned}$$

## Compound Interest

Compound interest investments or loans add the interest from one compounding period to the previous principal and use the sum as the principal for the next compounding period.

Compounding Period is the time interval after which the compound interest is calculated.

In an alternate dimension, Aayush deposits his \$1000 into a GIC that earns 3% compound interest annually. Each year, the interest is put back into the GIC.

Compounding Period	Principal for the Period	Interest Calculation	Amount at the End of the Period
1	\$ 1000	\$ 1000 (0.03) = \$ 30	\$ 1030
2	\$ 1030	\$ 1030 (0.03) = \$ 30.90	\$ 1060.90
3	\$ 1060.90	\$ 1060.90 (0.03) = \$ 31.83	\$ 1092.73
4	\$ 1092.73	\$ 32.782	\$ 1125.51

Examine the pattern in the last column, is this a arithmetic/geometric sequence/series?

Geometric sequence

$$t_n = a(r)^{n-1}$$

The compound interest formula is:

$$A = P(1+i)^n \quad (\text{modified GS formula})$$

$A$  is the amount

$P$  is the Principal

$i$  represents the interest rate per compounding period, expressed as a decimal

$n$  represents the number of compounding periods

**Example 2** To buy a new guitar, Joseph borrows \$650, which he plans to repay in 5 years. The bank charges 12% per annum, compounded annually.

a) Determine the amount that Joseph must repay.

$$A = 650 (1.12)^5$$

$$= 1145.52$$

b) How much interest will Joseph have to pay?

$$\$1145.52 - \$650 = \$495.52$$

c) Compare this to the amount of interest he would have to pay if the bank charged simple interest.

$$A = \$650 + (650 \times 0.12) \times 5 = \$1040$$

$$\$1040 - \$650 = \$390$$

### Vary the Compounding Period

Financial institutions often use other compounding periods rather than annual.

Frequency of Compounding	Number of Times Interest is Added During the Year
Annual	1
Semi-annual	2 (every 6 months)
Quarterly	4 (every 3 months)
Monthly	12 (every month)
Bi-weekly	26 (every 2 weeks)
Daily	365

**Example 3** Referring to Example 2, what will be the impact on the interest Joseph pays if the interest is compounded

a) Semi-annually?

$$n = \text{period} = 5 \text{ years} \times 2 = 10$$

$$i = \frac{0.12}{2} = 0.06$$

$$A = 650 (1.06)^{10}$$

$$= 1164.05$$

b) Monthly?

$$n = 5 \text{ years} \times 12 = 60$$

$$i = \frac{0.12}{12} = 0.01$$

$$A = 650 (1.01)^{60} \\ = 1180.85$$

#### Example 4

Nicole M. is starting a business. She applies for an \$8000 loan, which she plans to repay in 4 years. She is told by the loan officer that the amount payable when the loan is due is \$11 501.24. What rate of interest, compounded annually, is being charged?

$$A = P(1+i)^n$$

$$\frac{11501.24}{8000} = \frac{8000(1+i)^4}{8000}$$

$$\sqrt[4]{1.438} = \sqrt[4]{(1+i)^4}$$

$$\sqrt[4]{1.438} = 1+i$$

$$1.095 - 1 = i$$

$$i = 0.095 \text{ or } 9.5\%$$

∴ She is being charged 9.5%

#### Example 5

Mathushan wants to buy a car in 5 years. He estimates that he needs \$12 000 to purchase a decent car. How much does he need to put in a savings account now, if the account pays 4.1% interest compounded monthly?

$$A = P(1+i)^n$$

$$12000 = P(1+0.003417)^{60}$$

$$\frac{12000}{1.2271} = \frac{P \cdot 1.2271}{1.2271}$$

$$\$9779.15 = P$$

$$n = 5 \text{ years} \times 12 \text{ months} = 60$$

$$i = 0.041 \div 12 \text{ months} = 0.003417$$

∴ He needs to deposit

\$ 9779.15

Homework: pg. 423 #(1, 2), 5, 10  
pg. 433 #(1, 2), 4ac, 5ac, 8, 9, 13  
pg. 441 #1, 2, 3, 7, 14