MCR3U June 10

Ms. Kueh

Future Value of Annuities

* D2L Online Quiz open until Sunday 11 pm.

When people invest, they usually do not simply deposit one lump sum and wait several years for it to earn interest. Most investors make **regular payments,** often deducted directly from their pay checks. These type of investment are called annuities.

R ← **Regular Payment** – payments of equal value made at equal time periods e.g.) $500 /month

= equal deposits or payments

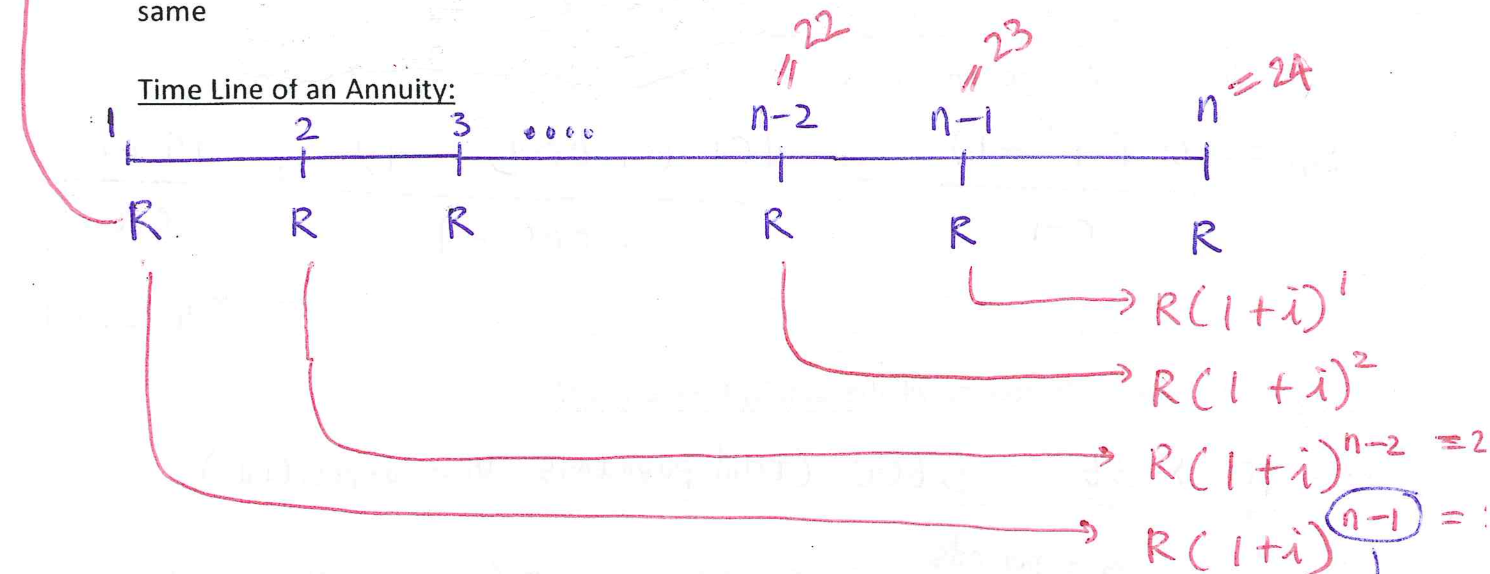
Annuity – a sum of money paid as a series of regular payments + interest

Ordinary annuity – an annuity for which the payments are made at the end of each payment period

same

Simple Annuity – an annuity for which the compounding and payment periods are the same

Time Line of an Annuity:



exponent indicates how many periods are still left the money will accum interest for you

The amount, A, of the annuity can be determined by adding the amounts of all the payments.

$$R + R(1+i) + R(1+i)^2 \cdots R(1+i)^{n-1}$$

$\times(1+i)$ $\times(1+i)$ ⟶ Geometric Series (compound i)

The formula for this sum is:

$$A = \frac{R\left((1+i)^n - 1\right)}{i} \quad \text{or} \quad S_n = \frac{a(r^n - 1)}{r-1}$$

A is the total amount at the time of the last payment (Also seen as Future Value)

R represents the regular payment in dollars

i represents the interest rate per compounding period, expressed as a decimal

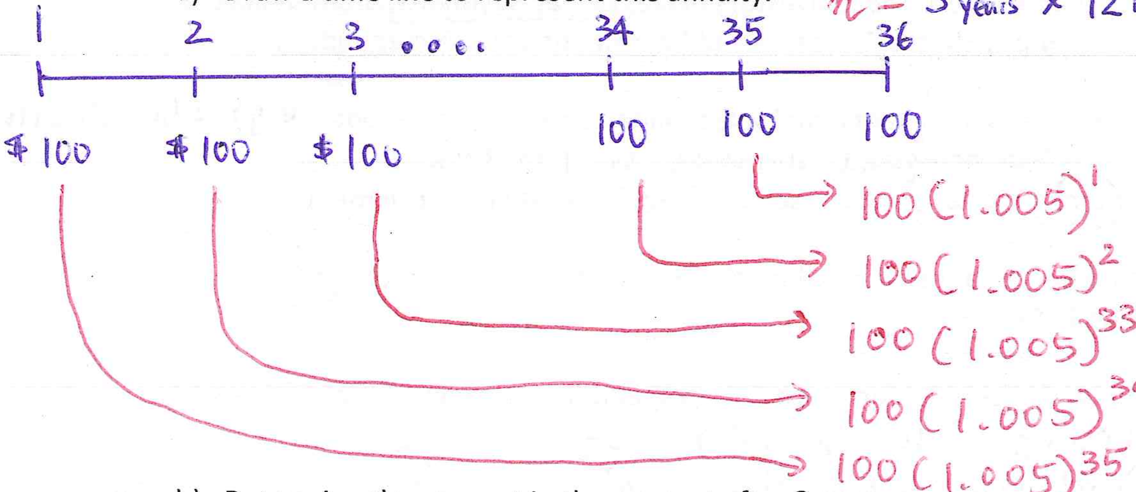
n represents the number of compounding periods

Example 1 ordinary annuity

At the end of every month, Katherine deposits $100 in an account that pays 6% per year, compounded monthly. She does this for 3 years.

a) Draw a time line to represent this annuity.

$n = 3 \text{ years} \times 12 \text{ months} = 36$ periods



$i = \frac{0.06}{12} = 0.005$

b) Determine the amount in the account after 3 years.

$$S_n = \frac{a(r^n - 1)}{r-1} = \frac{100(1.005^{36} - 1)}{1.005 - 1} = \frac{19.66805}{0.005} = 3933.61$$

c) How much interest will the annuity have earned?

$100 \times 36 = 3600$ (total payments you deposited)

↑ R ↑ n = periods

$3933.61 - 3600 = 333.61$

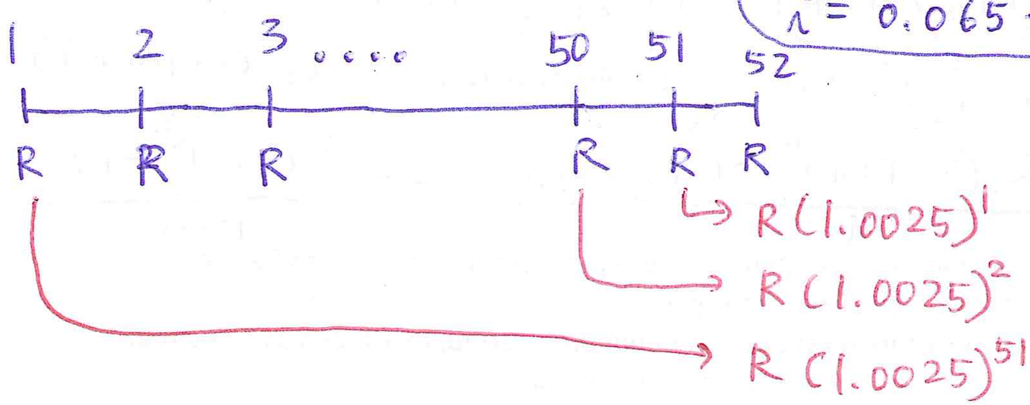
Example 2 $6000

Hannah needs ~~$4000~~ for university tuition when she graduates in 2 years. She plans to make deposits into an account that earns 6.5% per year, compounded bi-weekly.

a) Draw a time line to represent this annuity.

$n = 2 \text{ years} \times 26 = 52$ periods

$i = 0.065 \div 26 = 0.0025$



$R = ?$

$S_n = \$6000$

$r = 1 + i$

b) How much should she deposit bi-weekly?

$$S_n = \frac{a(r^n - 1)}{r-1} \qquad A = \frac{R((1+i)^n - 1)}{i}$$

$$6000 = \frac{a(1.0025^{52} - 1)}{(1.0025 - 1)}$$

$$6000 = \frac{a \cdot 0.13864}{0.0025}$$

$$6000 = a \cdot 55.456$$

$$\frac{6000}{55.456} = a$$

$$\therefore a = 108.19$$

Example 3 Vary the Conditions of an Annuity

Amir plans to invest \$2600 each year at 6% per year, compounded annually, for the next 15 years. Compare the effects on the final amount if the deposits are made and compounding periods are

a) annual

b) quarterly → 4 times a year → $R = 2600 \div 4 = 650\$$

c) monthly

d) weekly → $R = 2600 \div 52 = 50\$$

Do this on a separate sheet of paper if there is not enough room.

b) $n = 15 \text{ years} \times 4 = 60$ periods

$i = 0.06 \div 4 = 0.015$

$$S_n = \frac{a(r^n - 1)}{r-1} = \frac{650\left[(1.015)^{60} - 1\right]}{0.015}$$

$$= \frac{650 \cdot 1.44322}{0.015} = \$62{,}539.52$$

d) $n = 15 \text{ years} \times 52 \text{ weeks} = 780$

$i = 0.06 \div 52 = 0.00115385$

$$S_n = \frac{50\left[(1.00115385)^{780} - 1\right]}{0.00115385} = \$63194.29$$

Difference is \$654.77

$\therefore$ As a lender → choose "weekly" compounding periods.

borrower → // "annual" // //

Conclusions:

Homework: pg. 453 #2-5, 8-10 **Extension** #15