June 11

June 4, 2010

Present Value

Present Value is the principal invested or borrowed today to result in a given future amount, with given interest and time conditions

Future Value is the amount that a principal invested or borrowed will grow to, with given interest and time conditions

Using the Compound interest formula, we can say the Amount is the Future Value, and the Principal is the Present Value:

$$A = P(1+i)^n \rightarrow FV = PV(1+i)^n \rightarrow \frac{FV}{(1+i)^n} = PV$$

compound interest formula (Day 1)

$$\boxed{PV = \frac{FV}{(1+i)^n}}$$

PV is the present value
FV is the future value
i represents the interest rate per compounding period, expressed as a decimal
n represents the number of compounding periods

Example 1 Calculate the Present Value

Karlyn wants to invest money today to have $1000 (// FV) in 6 years. If she invests her money at 5.75% per year, compounded quarterly, how much does she need to invest?

$n = 6 \text{ years} \times 4 = 24 \text{ periods}$

$i = 5.75 \div 100 \div 4 = 0.0143$

$$PV = \frac{\$1000}{(1+0.0143)^{24}}$$

$$PV = \frac{\$1000}{1.40853} = \$709.96$$

∴ She needs to invest $709.96 today to gain $1000 6 years later.

Example 2 Determine the Number of Compounding Periods

Avery invests $250 at 6% per year, compounded monthly. When the account is closed, its value will be $317.62. How long will Avery's money be invested?

$n = x \times 12 = 12x$

$i = 6 \div 100 \div 12 = 0.005$/month

$PV = \$250$

$FV = \$317.62$

$$PV = \frac{FV}{(1+i)^n}$$

$$\frac{250}{1} = \frac{317.62}{(1+0.005)^{12x}}$$

$$(1.005)^{12x} \times 250 = 317.62$$

$$(1.005)^{12x} = \frac{317.62}{250}$$

$$(1.005)^{12x} = 1.27048$$

$12x = 48$

$\therefore x = 4$

∴ Avery will have to invest $250 today for 4 years

$*1.005^{25} = 1.13$

$*1.005^{24} = 1.127$

$*1.005^{50} = 1.283$

$*1.005^{48} = 1.2705$

2. The sum of the first n terms of a sequence is $n(n+1)(n+2)$. The sequence is not necessarily arithmetic or geometric. Find the 10^{th} term of the sequence.

MCR3U June 11

Ms. Kueh

Present Value of Annuities

Recall: Find the amount that Ms. Kueh needs to invest now, to buy a ~~grand piano~~ Camry (Toyota) costing \$30 000 in 10 years if she can get a 13% interest rate, compounded bi-weekly.

$n = 26 \times 10 = 260$

$i = \frac{0.13}{26} = 0.005$

$A = P(1+i)^n$

$30{,}000 = P(1+0.005)^{260}$

$\frac{30{,}000}{(1.005)^{260}} = P$

$\therefore P = \$8\,202.48$

The amount Ms. Kueh needs to invest now is called **present value.**

We could also rearrange our formula to create the "present value formula" instead of rearranging after we sub the numbers in.

$A = P(1+i)^n$ — A ↑ FV, P ↑ PV

$PV = \frac{FV}{(1+i)^n}$

Definitions:

The present value of an annuity is the total amount required to finance a series of regular withdrawals over a specific period of time.

Future Value of Annuity problems were posed in which regular payments are made into an account that grows to a large future amount.

If I make \$500/month deposit every month until 65 years old, how much money will I receive at 65 years old?

-for example, saving money every month for retirement.

lump sum amount

Present Value of Annuity problems will have regular withdrawals made from an account that begins with a large balance.

How much money do you have to deposit today to withdraw \$2500/month until you die?

-for example, after you have retired, withdrawing money every month for the rest of your life (you would have to estimate the number of years you expect to live!)

-after having saved for University/College, withdrawing money every month to pay tuition and living expenses

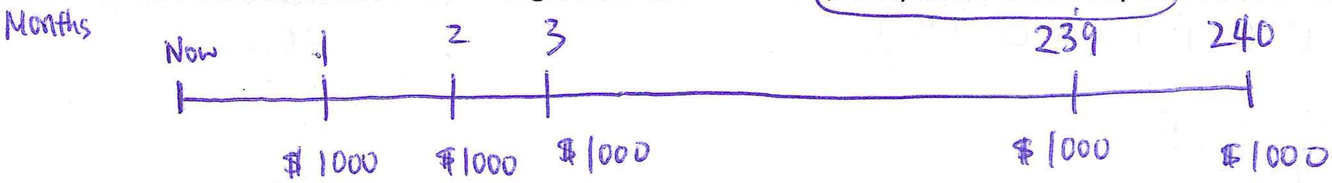
-mortgages are also present value of annuity problems, starting with a large debt, and slowly paying down the debt (decreasing the debt) over time.

You are buying \$500,000 house today, and you must borrow the money from the bank. What will be your regular payment?

To determine the present value required to finance a series of withdrawals, it is necessary to calculate the present value of each withdrawal using the present value formula:

Time Line for the Present Value of an Annuity:

Anna is retiring, and she needs to take out \$1000 every month for 20 years. How much money does she need **now** if she can get 9% annual interest, compounded monthly?



$n = 20 \text{ years} \times 12 \text{ months} = 240$

$i = 9 \div 100 \div 12 = 0.0075$

$R = \$1000$

$PV = ?$

$$PV = \frac{\$1000\left[1-(1+0.0075)^{-240}\right]}{0.0075} = \frac{\$1000 \times 0.833587}{0.0075}$$

$$= \$111{,}144.95$$

So the large amount of money required to finance time n is:

∴ She needs to invest \$111,144.95 today.

Is this an arithmetic/geometric sequence/series?

The formula for this sum is:

$$PV = \frac{R\left[1-(1+i)^{-n}\right]}{i}$$

< Present Value Annuity Formula >

PV represents the present value (lump sum)	***i*** represents the interest rate per compounding period, expressed as a decimal
R represents the regular withdrawal in dollars	***n*** represents the number of compounding periods

or payment (mortgage)

Example 1 Present Value of an Annuity

Shawna is putting her summer earnings into an annuity from which she can draw living expenses while she is at university. She will need to withdraw $\boxed{\$900 \text{ per month}}$ for 8 months. Interest is earned at a rate of 6% compounded monthly.

$R = \$900$

a) Draw a time line to represent this annuity.

$n = 8$, $\quad i = 6 \div 100 \div 12 \text{ months} = 0.005 \quad PV = ?$

$$PV = \frac{\$900\left[1-(1.005)^{-8}\right]}{0.005} = \frac{\$900 \cdot 0.039115}{0.005}$$

$$= \$7040.66$$

b) ∴ She needs to invest \$7040.66 today to withdraw \$900 per month.

b) How much does Shawna need to invest at the beginning of the school year to finance the annuity?

Example 2 Determine the Regular Withdrawal

Fast forward 40 years. Ms. Kueh's life savings total \$300 000 when she decides to retire. She plans an annuity that will pay her quarterly for the next 30 years. If her account earns 5.2% annual interest, compounded quarterly, how much can Ms. Kueh withdraw each quarter?

$n = 30 \times 4 = 120$

$i = 0.052 \div 4 = 0.013$

$R = ?$

$PV = \$300{,}000$

$$0.013 \times 300{,}000 = \frac{R\left(1-(1.013)^{-120}\right)}{0.013} \times 0.0$$

$$\frac{300{,}000 \times 0.013}{1-(1.013)^{-120}} = R$$

∴ She can withdraw \$4950.87/quarter ∴ $R = \dfrac{3900}{0.18774} = \$4950.87$

Mortgages and Amortization Tables

Short term e.g) 1year

When people buy a house, car, or condominium, they must arrange for a loan or a mortgage.

→ Long term e.g) 30 years

Loans and Mortgages are agreements between a money lender and a borrower to finance a purchase. = Debts

Mortgages are usually paid in equal payments at equal time intervals, with payments that include both principal and interest.

Amortize – to repay the mortgage over a given period of time in equal payments at regular intervals. The period of time is known as the **amortization period**. $= n$

Down Payment – a payment representing a fraction of the price of something being purchased. For a house, down payments for a conventional mortgage are at least 20% of the purchase price. $500,000 × 20% = $ 100,000

For Mortgages, the government requires them to be **compounded semi-annually**, but most people pay down the mortgages monthly.

So, the equivalent monthly interest rate is:

$$i = \left(1 + \frac{annual\ rate}{2}\right)^{\frac{1}{6}} - 1$$

**Try to figure out why this formula works*

The mortgage loan is the Present Value of an ordinary annuity. So we will use the equation

$$PV = \frac{R[1-(1+i)^{-n}]}{i}$$

PVA formula

Example 1 Finding the mortgage and interest rate

Nick is buying a house for $196 500. He makes a down payment of 25% of the price and negotiates a mortgage at 7.5%, amortized over 20 years, for the balance of the price.

a) How much is Nick's mortgage? (= How much does he have to borrow?)

$ 196500 × 0.75 = $ 147,375

$R = \frac{\$147,375 \times 0.0062}{[1-(1.00625)^{-24}}$

$R = \frac{921.094}{0.7758}$

$R = \$1187.28$

b) What is the equivalent monthly interest rate?

$i = 0.075 \div 12$ months $= 0.00625$

c) What is the number of compounding periods?

$n = 20$ years × 12 months = 240 periods